Separation Science and Technology
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713708471

# A Study of the Separation Efficiency in the Concentric-Tube Countercurrent Separation Process under Generalized Linear Applied Fields and with Recycles at Both Ends

Shau-Wei Tsai[a]; Ho-Ming Yeh[a]
[a] DEPARTMENT OF CHEMICAL ENGINEERING, NATIONAL CHENG KUNG UNIVERSITY, TAINAN, TAIWAN

**To cite this Article** Tsai, Shau-Wei and Yeh, Ho-Ming(1986) 'A Study of the Separation Efficiency in the Concentric-Tube Countercurrent Separation Process under Generalized Linear Applied Fields and with Recycles at Both Ends', Separation Science and Technology, 21: 10, 1141 — 1154
**To link to this Article: DOI:** 10.1080/01496398608058403
**URL:** http://dx.doi.org/10.1080/01496398608058403

SEPARATION SCIENCE AND TECHNOLOGY, 21(10), pp. 1141-1154, 1986

# A Study of the Separation Efficiency in the Concentric-Tube Countercurrent Separation Process under Generalized Linear Applied Fields and with Recycles at Both Ends

---

SHAU-WEI TSAI and HO-MING YEH

DEPARTMENT OF CHEMICAL ENGINEERING
NATIONAL CHENG KUNG UNIVERSITY
TAINAN, TAIWAN, REPUBLIC OF CHINA

## Abstract

The mathematical model for the separation of binary mixtures has been extended to a concentric-tube continuous-contact countercurrent column under generalized linear external fields and with recycles at both ends. An analytical solution is obtained by use of the orthogonal expansion method. Numerical results for separation in a thermal diffusion column are also illustrated.

## INTRODUCTION

The problem of heat or mass transfer for fully developed flow in a conduit with negligible axial conduction or diffusion is well known as the Graetz problem. The governing equation of this problem is usually transformed into an eigenvalue problem by means of the separation of variables method. The resultant eigenfunction may be found numerically, usually by assuming a power series expansion, by the Runge-Ketta integration scheme, or even in terms of well-known functions such as the confluent hypergeometric function. Recently, many investigators have extended the classical Graetz problem to different geometries and a variety of boundary conditions (*1, 3, 4*). Yet, the velocity distribution they treated does not change sign over the interval in question. Accordingly, only a positive set of eigenvalues exists.

0149-6395/86/2110-1141$3.50/0

In order to obtain higher separation efficiency, many separation processes in chemical engineering have been performed in countercurrent operation and with refluxes at the ends. Typical examples are distillation or extraction in a rectifying column, continuous menbrane separation, thermal diffusion, countercurrent centrifuges, mass diffusion, etc. For the equilibrium-stage operation, many methods, such as the Sorel method, are adequate to solve the problem. For diffusion-rate operation, a thorough analysis is still necessary. The difficulties arise due to the changing sign of the velocity profile and because the concentrations at both ends are unspecified *a priori*.

For homogeneous mixtures, separation may be reached by the use of such external fields as force, pressure, and temperature. In a previous work (*8*) the separation theory in a continuous-contact countercurrent separation process under constant applied fields and with refluxes at the ends was derived. It is the purpose of this work to extend the theory of the above problems under the influence of generalized linear applied fields.

## THEORETICAL FORMULATION

Consider an ideal concentric-tube countercurrent separation column with binary feed introduced from some intermediate position of the column. As shown in Fig. 1, the column is composed of enriching and stripping sections. Fully developed fluid flows countercurrently with internal or external refluxes at the ends. Products are withdrawn continuously with mass flow rates $\sigma_t$ and $\sigma_b$, respectively, from the ends.

The theory is developed on the basis of the following assumptions to predict the product concentrations:

(1) Purely laminar flow exists in both sections.
(2) Physical properties are constant, and no bulk flow exists in the horizontal direction.
(3) End effects and axial diffusion are negligible.
(4) External fields, such as temperature and electric fields as used by Crosser et al. (*2*), are applied uniformly in the radial direction to cause the desired product concentrated to the left and to the top of the column.

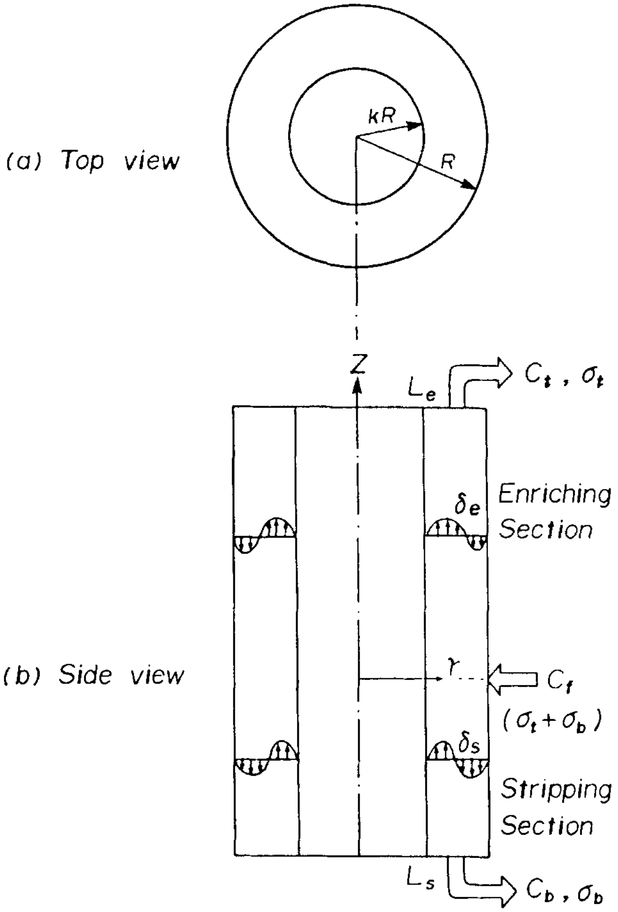

FIG. 1. Schematic diagram of a continuous-contact countercurrent separation process with generalized linear applied fields and with recycles at both ends.

Therefore, the mass flux in the radial direction can be formulated as

$$J_{r,e} = \frac{\rho D}{R}\left\{\frac{\partial C_e}{\partial \eta} - g(\eta)C_e - f(\eta)\right\} \tag{1}$$

in which the linear function of concentration $[g(\eta)C_e + f(\eta)]$ is assumed. With constant driving forces, it is well known that the flux due to forced diffusion is a linear function of concentration, while that of thermal and pressure diffusion is a quadratic function. However, the quadratic form of concentration can be approximately linearized, as in thermal diffusion (*5*, *6*). Accordingly, the separation processes due to force, pressure, and thermal diffusions may be regarded as those present in a linear field.

The mass balance for the enriching section yields

$$U_e(\eta)\frac{\partial C_e}{\partial \zeta} = \frac{1}{\eta}\frac{\partial}{\partial \eta}\left\{\eta\left[\frac{\partial C_e}{\partial \eta} - g(\eta)C_e - f(\eta)\right]\right\} \tag{2}$$

where the dimensionless coordinates and dimensionless velocity are defined as

$$\zeta = z/L_e, \qquad \eta = r/R, \qquad U_e(\eta) = \frac{R^2V_e(r)}{DL_e} \tag{3}$$

The boundary conditions for solving Eq. (2) are

$$\partial C_e/\partial \eta = g(\eta)C_e + f(\eta), \qquad \text{at } \eta = k, 1 \tag{4}$$

$$C_e = C_t, \qquad \text{at } \zeta = 1 \tag{5}$$

where Eq. (4) is due to impermeable walls for mass transfer. Since it is imperative to have a mixing zone at the end due to reflux, we have imposed Eq. (5) on the boundary. However, $C_t$ is still unknown.

For the stripping section, all above equations are valid except that the subscript $e$ is replaced by $s$ and Eq. (5) by Eq. (6):

$$C_s = C_b, \qquad \text{at } \zeta = -1 \tag{6}$$

Just as for $C_t$ in the enriching section, $C_b$ is unknown at present.

## SOLUTION OF CONCENTRATION FIELDS

In order to transform the governing partial differential equation into a Sturm-Liouville equation, we may assume the concentration profile in the enriching section as

$$C_e(\eta, \zeta) = \sum_{m=0}^{\infty} S_{e,m}F_{e,m}(\eta)G_{e,m}(\zeta) + Y(\eta) \tag{7}$$

Substituting Eq. (7) into Eq. (2) yields

$$G_{e,m}(\zeta) = \exp\,[-\lambda_{e,m}(1 - \zeta)] \tag{8}$$

$$Y(\eta) = \left\{ \int_k^{\eta} f(\eta) \exp\left[ - \int_k^{\eta} g(\eta)d\eta \right] d\eta \right\} \exp\left[ \int_k^{\eta} g(\eta)d\eta \right] \tag{9}$$

$$\left\{\eta F'_{e,m}(\eta) - \eta g(\eta)F_{e,m}(\eta)\right\}' - \lambda_{e,m}\eta U_e(\eta)F_{e,m}(\eta) = 0 \tag{10}$$

$$F'_{e,m}(\eta) = g(\eta)F_{e,m}(\eta), \qquad \text{at } \eta = k,\ 1 \tag{11}$$

where the primes on the eigenfunction denote differentiation with respect to $\eta$. We have chosen the form of Eq. (8) for convenience in the later derivation of the expansion coefficient.

Many methods have been proposed to solve the eigenvalue and associated eigenfunction. For example, one may use a power series expansion method, a special function such as a confluent hypergeometric function (or Kummer function), or an integration method. For a generalized velocity distribution, it is appropriate to use an integration scheme such as the Runge-Kutta method. Since Eqs. (10) and (11) are a special case of Sturm-Liouville equation for which the velocity profile changes sign over the interval in question, there may exist both positive and negative sets of real eigenvalues that have limiting points $+\infty$, and $-\infty$, respectively. For the stripping section, all the above equations and descriptions are still valid except that the subscript $e$ is replaced by $s$ and Eq. (8) by Eq. (12):

$$G_{s,m}(\zeta) = \exp\,[\lambda_{s,m}(1 + \zeta)] \tag{12}$$

Now, only $C_t$, $C_b$, $S_{e,m}$, and $S_{s,m}$ are still undetermined.

Since zero is also the eigenvalue for each section, one can solve the associated eigenfunction from Eqs. (10) and (11) as $\exp\,[\int_k^{\eta} g(\eta)d\eta]$ for both sections. Therefore, the concentration distribution, say for the enriching section, can be rewritten as

$$C_e(\eta, \zeta) = S_{e,o} \exp\left[ \int_k^{\eta} g(\eta)d\eta \right] + \sum_{m=1}^{m} S_{e,m}F_{e,m}(\eta)G_{e,m}(\zeta) + Y(\eta) \tag{13}$$

It is easy to find the orthogonality conditions for both sections with weight functions $\eta U_e(\eta) \exp\,[-\int_k^{\eta} g(\eta)d\eta]$ and $\eta U_s(\eta) \exp\,[-\int_k^{\eta} g(\eta)d\eta]$, respectively. Accordingly, by applying the concentration distributions and orthogonality conditions for both sections, the boundary conditions at the ends, and integrating for $\eta$ from $k$ to 1 yields

$$S_{e,o} = \left\{ \int_k^1 [C_t - Y(\eta)]\eta U_e(\eta)d\eta \right\} \Big/ \left\{ \int_k^1 \eta U_e(\eta) \exp\left[ \int_k^\eta g(\eta)d\eta \right] d\eta \right\} \tag{14}$$

$$A_{1e,m}C_t = A_{2e,m}S_{e,m} + A_{3e,m} \tag{15}$$

$$A_{1s,m}C_b = A_{2s,m}S_{s,m} + A_{3s,m} \tag{16}$$

where

$$A_{1e,m} = \frac{1}{\lambda_{e,m}} \int_k^1 \eta g(\eta) \left\{ F_{e,m}(\eta) \exp\left[ - \int_k^\eta g(\eta)d\eta \right] \right\}' d\eta \tag{17}$$

$$A_{2e,m} = \left\{ \eta \left[ F_{e,m}(\eta) \frac{\partial F'_{e,m}(\eta)}{\partial \lambda_{e,m}} - F'_{e,m}(\eta) \frac{\partial F_{e,m}(\eta)}{\partial \lambda_{e,m}} \right] \exp\left[ - \int_k^\eta g(\eta)d\eta \right] \right\} \Bigg|_k^1 \tag{18}$$

$$A_{3e,m} = \frac{-1}{\lambda_{e,m}} \int_k^1 \eta [F'_{e,m}(\eta) - g(\eta)F_{e,m}(\eta)] \left\{ Y(\eta) \exp\left[ - \int_k^\eta g(\eta)d\eta \right] \right\}' d\eta \tag{19}$$

$$S_{s,o} = S_{e,o}|_{e\to s}, \quad A_{1s,m} = A_{1e,m}|_{e\to s}, \quad A_{2s,m} = A_{2e,m}|_{e\to s}, \quad A_{3s,m} = A_{3e,m}|_{e\to s} \tag{20}$$

Define the difference of average concentrations for both sections by

$$C_t - C_i = \left\{ \int_k^{\delta_e} \eta U_e(\eta)[C_e(\eta, 1) - C_e(\eta, 0)]d\eta \right\} \left\{ \left[ \int_k^{\delta_e} \eta U_e(\eta)d\eta \right]^{-1} - \left[ \int_{\delta_e}^1 \eta U_e(\eta)d\eta \right]^{-1} \right\} / 2 \tag{21}$$

$$C_i - C_b = \left\{ \int_k^{\delta_s} \eta U_s(\eta)[C_s(\eta, 0) - C_s(\eta, -1)]d\eta \right\} \left\{ \left[ \int_k^{\delta_s} \eta U_s(\eta)d\eta \right]^{-1} - \left[ \int_{\delta_s}^1 \eta U_s(\eta)d\eta \right]^{-1} \right\} / 2 \tag{22}$$

where $C_i$ denotes the average concentration at the feed position in the column. The quantities $\delta_e$ and $\delta_s$ are the positions where $U_e(\eta)$ and $U_s(\eta)$

begin to change sign, respectively. The reasons for such definitions are explained in the Appendix.

Combining Eqs. (21) and (22) to eliminate $C_i$ and applying Eqs. (10), (11), (13), and those for the stripping section results in

$$\begin{aligned}\Delta = C_t - C_b = &\left\{ \sum_{m=1}^{\infty} S_{e,m}\delta_e[F'_{e,m}(\delta_e) - g(\delta_e)F_{e,m}(\delta_e)][1 - \exp(-\lambda_{e,m})]\right\} \\ &\left\{\left[\int_k^{\delta_e} \eta U_e(\eta)d\eta\right]^{-1} - \left[\int_{\delta_e}^{1} \eta U_e(\eta)d\eta\right]^{-1}\right\} / \left\{2\lambda_{e,m}\right\} + \\ &\left\{ \sum_{m=1}^{\infty} S_{s,m}\delta_s[F'_{s,m}(\delta_s) - g(\delta_s)F_{s,m}(\delta_s)][\exp(\lambda_{s,m}) - 1]\right\} \\ &\left\{\left[\int_k^{\delta_s} \eta U_s(\eta)d\eta\right]^{-1} - \left[\int_{\delta_e}^{1} \eta U_s(\eta)d\eta\right]^{-1}\right\} / \{2\lambda_{s,m}\}\end{aligned} \tag{23}$$

A material balance around the whole column gives

$$(\sigma_t + \sigma_b)C_f = \sigma_t C_t + \sigma_b C_b \tag{24}$$

Theoretically, all nonzero expansion coefficients, $C_t$, and $C_b$ may be solved from simultaneous linear algebraic Eqs. (15), (16), (23), and (24). It is obvious that the eigenvalue of zero has no influence on the determination of $C_t$ and $C_b$, as shown in Eq. (23). Therefore, when $C_t$, $C_b$, and all nonzero expansion coefficients are obtained, one may find the concentration at any point, say from Eqs. (13) and (14), for the enriching section. Moreover, it is easy to show that the negative set of eigenvalues for $\lambda_{e,m}$ and the positive set of eigenvalues for $\lambda_{s,m}$ can be neglected in order to make the values obtained from Eqs. (21)–(23) convergent.

One may also reduce the above derivations to those obtained in the previous work (*8*). When constant external fields are applied, i.e., $g(\eta) = 0$, one may obtain $A_{1e,m} = A_{1s,m} = 0$ by inspection from Eqs. (17) and (20). Therefore, all nonzero expansion coefficients are given directly from Eqs. (15) and (16) without solving a set of simultaneous algebraic equations. Then, the concentrations of product are obtained from Eqs. (23) and (24), simultaneously.

## COMPUTATION ASPECT

For the purpose of illustration, we will consider the separation of a benzene and *n*-heptane mixture in a concentric-tube thermogravitational thermal diffusion column, as shown in Fig. 2. The mass flux due to

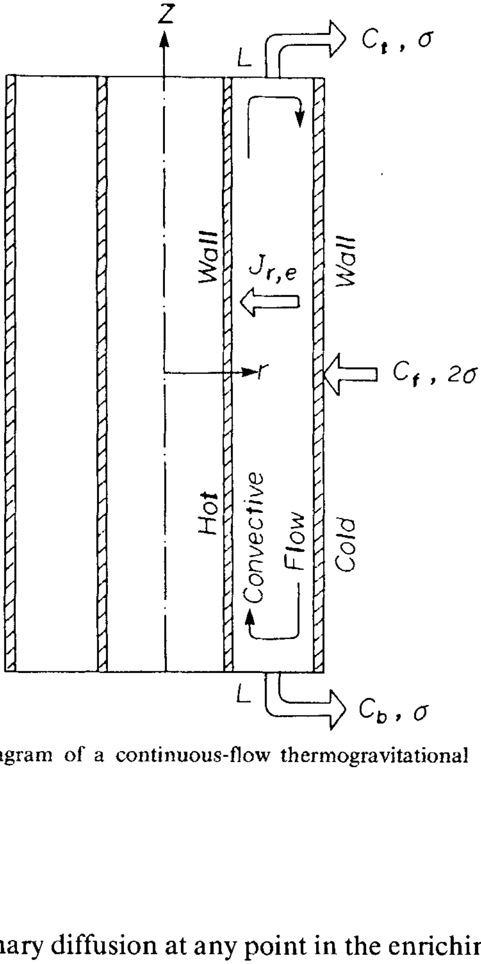

FIG. 2. Schematic diagram of a continuous-flow thermogravitational thermal diffusion column.

thermal and ordinary diffusion at any point in the enriching section may be expressed as (5)

$$J_{r,e} = \frac{\rho D}{R}\left[\frac{\partial C_e}{\partial \eta} - \frac{\alpha C_e(1 - C_e)}{\bar{T}}\frac{dT}{d\eta}\right] \tag{25}$$

where α is the thermal diffusion constant and $\bar{T}$ is the reference temperature of the mixture in the column under batch operation. The quadratic form of concentration may be linearized as

$$C_e(1 - C_e) = a + bC_e \tag{26}$$

The appropriate values of $a$ and $b$ may be determined by use of least-squares method, i.e.,

$$Q_{\min} = \begin{cases} \int_0^{2C_f} [C_e(1 - C_e) - (a + bC_e)]^2 \, dC_e, & \text{for } 0 \leqslant C_e \leqslant 0.5 \\ \int_{2C_f - 1}^{1} [C_e(1 - C_e) - (a + bC_e)]^2 \, dC_e, & \text{for } 0.5 \leqslant C_e \leqslant 1.0 \end{cases} \tag{27}$$

Hence, we obtain

$$\begin{aligned} &a = 2C_f^2/3, \; b = 1 - 2C_f; && \text{for } 0 \leqslant C_f \leqslant 0.5 \\ &a = (2C_f^2 + 2C_f - 1)/3, \; b = 1 - 2C_f; && \text{for } 0.5 \leqslant C_f \leqslant 1.0 \end{aligned} \tag{28}$$

One may furthermore assume:

(1) The temperature distribution is determined by conduction in the radial direction only. This is due to the small annulus space in the thermal diffusion column.
(2) The velocity distribution may be solved from the equations of motion and energy (7).
(3) Feed is introduced at the center of the column and product is withdrawn with the same flow rate, i.e., $\sigma_e = \sigma_s = \sigma$. Accordingly, one obtains

$$f(\eta) = \frac{a\alpha(\Delta T)}{\eta \bar{T} \ln k} \tag{29}$$

$$g(\eta) = \frac{b\alpha(\Delta T)}{\eta \bar{T} \ln k} \tag{30}$$

$$\bar{T} = T_1 - \phi - \frac{\Delta T}{\ln k} \tag{31}$$

$$\phi = \frac{-(1 - 4k^2 + 3k^4 - 4k^4 \ln k)(\Delta T)}{4[(1 - k^4) \ln k + (1 - k^2)^2]} \tag{32}$$

$$U_e(\eta) = \frac{-\beta g R^4}{LD\mu} \left[ (\phi + \omega_e) \left( \eta^2 - \frac{(k^2 - 1) \ln \eta}{\ln k} - 1 \right) - \frac{(\Delta T)}{4 \ln k} (k^2 \ln \eta - \eta^2 \ln k) \right] \tag{33}$$

TABLE 1. Eigenvalues and Related Coefficients

| $\sigma$ (g/min) | $m$ | $\delta_e$ | $\lambda_{e,m}$ | $\frac{A_{2e,m}}{A_{1e,m}}$ | $\frac{A_{3e,m}}{A_{1e,m}}$ | $F_{e,m}(\delta_e)$ | $F'_{e,m}(\delta_e)$ |
|---|---|---|---|---|---|---|---|
| $5.0^{(-2)}$ | 1 | $9.80106^{(-1)}$ | $7.50864^{(-1)}$ | 1.31045 | $-8.33333^{(-3)}$ | 1.00477 | 1.91080 |
|  | 2 | $9.80106^{(-1)}$ | $1.54206^{(2)}$ | $1.58382^{(2)}$ | $-8.33333^{(-3)}$ | $-6.60400^{(-1)}$ | $-9.84217^{(1)}$ |
| $1.0^{(-1)}$ | 1 | $9.80171^{(-1)}$ | $9.26362^{(-1)}$ | 1.59851 | $-8.33333^{(-3)}$ | $9.93922^{(-1)}$ | 2.95005 |
|  | 2 | $9.80171^{(-1)}$ | $1.58913^{(2)}$ | $1.59931^{(2)}$ | $-8.33333^{(-3)}$ | $-6.52943^{(-1)}$ | $-9.94222^{(1)}$ |
| $5.0^{(-1)}$ | 1 | $9.80692^{(-1)}$ | 2.34069 | 3.69000 | $-8.33333^{(-3)}$ | $9.17061^{(-1)}$ | $1.04943^{(1)}$ |
|  | 2 | $9.80692^{(-1)}$ | $1.72268^{(2)}$ | $1.63962^{(2)}$ | $-8.33333^{(-3)}$ | $-6.55554^{(-1)}$ | $-1.01995^{(2)}$ |
| 1.0 | 1 | $9.81343^{(-1)}$ | 4.16868 | 5.86791 | $-8.33333^{(-3)}$ | $8.41501^{(-1)}$ | $1.83541^{(1)}$ |
|  | 2 | $9.81343^{(-1)}$ | $1.95753^{(2)}$ | $1.70159^{(2)}$ | $-8.33333^{(-3)}$ | $-6.50309^{(-1)}$ | $-1.06176^{(2)}$ |
| 3.0 | 1 | $9.83949^{(-1)}$ | $1.30903^{(1)}$ | $1.15024^{(1)}$ | $-8.33333^{(-3)}$ | $6.83473^{(-1)}$ | $3.91879^{(1)}$ |
|  | 2 | $9.83949^{(-1)}$ | $3.43619^{(2)}$ | $1.97197^{(2)}$ | $-8.33333^{(-3)}$ | $-6.33357^{(-1)}$ | $-1.25543^{(2)}$ |

[a] $9.80106^{(-1)}$ means $9.80106 \times 10^{-1}$.

$$\omega_e = \frac{2\mu\sigma \ln k}{\rho\pi R^4 \beta g[(1 - k^4)\ln k + (1 - k^2)^2]} \tag{34}$$

We may also obtain $\delta_e$ by letting $U_e(\eta)$ equal zero. For the stripping section, Eqs. (29)–(34) are still valid except that the subscript $e$ is replaced by $s$, and $\sigma$ in Eq. (34) is replaced by $-\sigma$.

Let the numerical values be as follows:

$$R = 1.6 \text{ cm}, L_e = L_s = 60 \text{ cm}, k = 0.96, T_1 = 288.5 \text{ K}, T_2 = 322.5 \text{ K},$$

$$\alpha = 1.2, g = 980 \text{ cm/s}^2, \mu = 4.48 \times 10^{-3} \text{ g/cm}\cdot\text{s}, \beta = 1.27 \times 10^{-3} \text{ g/K}\cdot\text{cm}^3,$$

$$\rho = 0.758 \text{ g/cm}^3, D = 2.12 \times 10^{-5} \text{ cm}^2\text{/s}$$

Using these values, the first two nonzero eigenvalues and related coefficients for $C_f = 0.1$ in both sections have been calculated by the Runge-Kutta integration method and the results are presented in Table 1 with the product rate $\sigma$ as parameter. The degree of separation and product concentrations are also evaluated with $\sigma$ and $C_f$ as parameters and are presented in Table 2. During the preparation of Table 2, only the first eigenvalue for each section is necessary. This is due to the large ratio of column length to column annulus.

with Mass Flow Rate σ as the Parameter for $C_f = 0.1^a$

| $S_{e,m}$ | $\delta_s$ | $\lambda_{s,m}$ | $\frac{A_{2s,m}}{A_{1s,m}}$ | $\frac{A_{3s,m}}{A_{1s,m}}$ | $F_{s,\mathrm{m}}(\delta_s)$ | $F'_{s,m}(\delta_s)$ | $S_{s,m}$ |
|---|---|---|---|---|---|---|---|
| $1.03578^{(-1)}$ | $9.79976^{(-1)}$ | $-1.49611^{(2)}$ | $9.02116^{(3)}$ | $-8.33333^{(-3)}$ | $3.70557^{(1)}$ | $-5.58970^{(3)}$ | $8.97141^{(-6)}$ |
| $8.57004^{(-4)}$ | $9.79976^{(-1)}$ | $-4.71443^{(2)}$ | $-6.04372^{(5)}$ | $-8.33333^{(-3)}$ | $9.06405^{(2)}$ | $-2.02157^{(5)}$ | $-1.33912^{(-7)}$ |
| $8.33776^{(-2)}$ | $9.79911^{(-1)}$ | $-1.51423^{(2)}$ | $9.53158^{(3)}$ | $-8.33333^{(-3)}$ | $3.89687^{(1)}$ | $-5.90513^{(3)}$ | $8.74844^{(-6)}$ |
| $8.33360^{(-4)}$ | $9.79911^{(-1)}$ | $-4.77101^{(2)}$ | $-6.62154^{(5)}$ | $-8.33333^{(-3)}$ | $9.89697^{(2)}$ | $-2.21612^{(5)}$ | $-1.25932^{(-7)}$ |
| $3.30351^{(-2)}$ | $9.79390^{(-1)}$ | $-1.66993^{(2)}$ | $1.51130^{(4)}$ | $-8.33333^{(-3)}$ | $5.95764^{(1)}$ | $-9.37657^{(3)}$ | $6.27058^{(-6)}$ |
| $7.43462^{(-4)}$ | $9.79390^{(-1)}$ | $-5.25717^{(2)}$ | $-1.42940^{(6)}$ | $-8.33333^{(-3)}$ | $2.07826^{(3)}$ | $-4.82111^{(5)}$ | $-6.62984^{(-8)}$ |
| $1.98339^{(-2)}$ | $9.78739^{(-1)}$ | $-1.89488^{(2)}$ | $2.84807^{(4)}$ | $-8.33333^{(-3)}$ | $1.08718^{(2)}$ | $-1.77288^{(4)}$ | $3.52110^{(-6)}$ |
| $6.83968^{(-4)}$ | $9.78739^{(-1)}$ | $-5.95986^{(2)}$ | $-4.16162^{(6)}$ | $-8.33333^{(-3)}$ | $5.83949^{(3)}$ | $-1.41482^{(6)}$ | $-2.40972^{(-8)}$ |
| $9.67908^{(-3)}$ | $9.76138^{(-1)}$ | $-3.30926^{(2)}$ | $8.74288^{(5)}$ | $-8.33333^{(-3)}$ | $2.79458^{(3)}$ | $-5.57725^{(5)}$ | $1.20480^{(-7)}$ |
| $5.64575^{(-4)}$ | $9.76138^{(-1)}$ | $-1.03786^{(3)}$ | $-1.47436^{(9)}$ | $-8.33333^{(-3)}$ | $1.77409^{(6)}$ | $-5.16304^{(8)}$ | $-7.14435^{(-11)}$ |

## DISCUSSION AND CONCLUSIONS

(1) The equation of separation for the separation efficiency in squared-off continuous-contact countercurrent separation columns with generalized linear applied fields and with recycles at both ends has been derived by using the orthogonal expansion method. The expansion coefficients for both sections are obtained by solving a set of simultaneous algebraic equations, which is quite different from the cases with constant applied field derived previously (*8*). In order to make the solution convergent, the negative set of eigenvalues for the enriching section and the positive set of eigenvalues for the stripping section should be omitted. Moreover, during the calculation of product concentrations, the eigenvalue of zero for each section is also neglected.

(2) A numerical example for the separation of a benzene and *n*-heptane mixture in a continuous thermogravitational thermal diffusion column has been illustrated. Two nonzero eigenvalues and related expansion coefficients for both sections with $C_f = 0.1$ are calculated and presented in Table 1 with the mass flow rate as the parameter. The degree of separation and product concentrations are also evaluated and presented in Table 2 with $C_f$ and σ as parameters. However, only the first eigenvalue for each section is necessary. This is due to the large ratio of column

TABLE 2

Degree of Concentration and Concentrations of Product with $\sigma$ as the Parameter for Various Feed Concentrations[a]

| | $C_f = 0.1$ | | | $C_f = 0.3$ | | | $C_f = 0.7$ | | | $C_f = 0.9$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma$ (g/min) | $C_t$ (%) | $C_b$ (%) | $\Delta$ (%) | $C_t$ (%) | $C_b$ (%) | $\Delta$ (%) | $C_t$ (%) | $C_b$ (%) | $\Delta$ (%) | $C_t$ (%) | $C_b$ (%) | $\Delta$ (%) |
| $5.0^{(-2)}$ | 12.75 | 7.25 | 5.50 | 35.85 | 24.15 | 11.70 | 75.85 | 64.15 | 11.70 | 92.75 | 87.25 | 5.50 |
| $1.0^{(-1)}$ | 12.50 | 7.50 | 5.00 | 35.35 | 24.65 | 10.70 | 75.35 | 64.65 | 10.70 | 92.50 | 87.50 | 5.50 |
| $5.0^{(-1)}$ | 11.36 | 8.64 | 2.72 | 32.94 | 27.06 | 5.88 | 72.94 | 67.06 | 5.88 | 91.36 | 88.64 | 2.72 |
| 1.0 | 10.81 | 9.19 | 1.62 | 31.74 | 28.26 | 3.48 | 71.74 | 68.26 | 3.48 | 90.81 | 89.19 | 1.62 |
| 3.0 | 10.30 | 9.70 | 0.60 | 30.64 | 29.36 | 1.28 | 70.64 | 69.36 | 1.28 | 90.30 | 89.70 | 0.60 |

[a] $5.0^{(-2)}$ means $5.0 \times 10^{-2}$.

length to column annulus in the thermal diffusion column.

(3) For the special case where the feed is introduced from the bottom of the column, the stripping section is missing. Hence, only the terms associated with the enriching section are retained. One may also follow the same procedures to obtain product concentrations. For the same reason, one may find the solution when the feed is introduced from the top of the column.

(4) For a column with a small or moderate ratio of column length to column annulus, the effects of mixing zones at the top, the bottom, and the feed position of the column on the separation efficiency may be large. Therefore, the effective column length will be the subject of further study.

## APPENDIX

If the integrations in Eqs. (21) and (22) are performed from *k* to 1, conflicting results are obtained:

$$\int_k^1 \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta = \int_k^1 \eta U_s(\eta)[C_s(\eta,0) - C_s(\eta,-1)]d\eta$$
$$= 0 \qquad \text{(A1)}$$

This is due to the facts that the velocity distributions have changed sign and boundary conditions. Consequently, we may define

$$C_t - C_i = \frac{1}{2}\left\{ \frac{\int_k^{\delta_e} \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta}{\int_k^{\delta_e} \eta U_e(\eta)d\eta} + \frac{\int_{\delta_e}^{1} \eta U_e(\eta)[C_e(\eta,1) - C_e(\eta,0)]d\eta}{\int_{\delta_e}^{1} \eta U_e(\eta)d\eta} \right\} \qquad \text{(A2)}$$

By combining Eqs. (A1) and (A2), Eq. (21) is obtained. It is easy to obtain Eq. (22) with the same procedure.

## SYMBOLS

| | |
|---|---|
| $C$ | weight fraction of Component 1 in binary mixture |
| $C_f$, $C_i$ | $C$ in feed stream and in the feed position of the column, respectively |
| $D$ | ordinary diffusivity |
| $g$ | gravitational acceleration |
| $J_r$ | mass flux in the $r$-direction |
| $k$ | ratio of the outer radius of the inner tube to the inner radius of the outer tube |
| $L$ | column length of each section |
| $R$ | inner radius of the outer tube |
| $r$ | coordinate in the radial direction |
| $S_m$ | expansion coefficient associated with eigenvalue $\lambda_m$ |
| $\tilde{T}$ | reference temperature in the thermal diffusion column |
| $T_1$, $T_2$ | temperatures of cold wall and hot wall, respectively, in the thermal diffusion column |
| $U$ | dimensionless velocity distribution |
| $V$ | velocity distribution |
| $z$ | coordinate in the axial direction |

### Greek Letters

| | |
|---|---|
| $\alpha$ | thermal diffusion constant |

| | |
|---|---|
| $\beta$ | thermal expansion coefficient |
| $\Delta$ | degree of separation, i.e., $C_t - C_b$ |
| $\Delta T$ | $T_2 - T_1$ |
| $\delta$ | position at which the velocity distribution changes sign |
| $\zeta$ | dimensionless coordinate in the axial direction |
| $\eta$ | dimensionless coordinate in the radial direction |
| $\lambda_m$ | eigenvalue |
| $\mu$ | viscosity of the mixture |
| $\rho$ | density of the mixture |
| $\sigma$ | mass flow rate of product |

## Subscripts

| | |
|---|---|
| $b$ | at the bottom of the column |
| $e$ | for the enriching section |
| $s$ | for the stripping section |
| $t$ | at the top of the column |

## Acknowledgment

The authors wish to express their thanks to the Chinese National Science Council for financial aid.

## REFERENCES

1. Y. Bayazitoglu and M. N. Ozisik, *Int. J. Heat Mass Transfer, 23,* 1399 (1980).
2. O. K. Crosser, J. E. Powers, and P. K. Prabhudesai, *AIChE J., 19,* 38 (1973).
3. V. D. Dang, *Chem. Eng. Sci., 33,* 1179 (1978).
4. E. J. Davis, *Can. J. Chem. Eng., 51,* 562 (1973).
5. W. H. Furry, R. C. Jones, and L. Onsager, *Phys. Rev., 55,* 1939 (1939).
6. H. M. Yeh and T. J. Chu, *Chem. Eng. Sci., 30,* 47 (1975).
7. H. M. Yeh, *Sep. Sci. Technol., 11,* 455 (1976).
8. H. M. Yeh, T. W. Tseng, and S. W. Tsai, *Ibid., 21,* 403 (1986).

*Received by editor March 10, 1986*